Materializing a binary hyperlens design

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We present a design of cylindrical hyperlenses made of a layered binary material. The design approach uses an improved effective medium theory to take account of radius-dependent effects due to curvature of material interfaces resulting in nonperiodically distributed thicknesses of the lens layers. The performance of this lens is compared versus the designs with periodically thick layers, which we showed in earlier papers. Detailed quantitative results analyzed for the lenses with the same number and starting order of layers prove better functioning of the lens designed with this approach. © 2009 American Institute of Physics. [DOI: 10.1063/1.3081403]

The recent advancement theory of hyperlenses and enhanced nanofabrication techniques have provided us with a fresh perspective on magnifying subwavelength imaging devices made of anisotropic materials with hyperbolic dispersion. In prefabrication design and experiments the required anisotropy was achieved through the fabrication of a cylindrical lamellar structure made of two distinct elementary materials (i.e., a binary metal-dielectric composite). A periodic distribution of metal-dielectric layers of a fixed thickness has been used. The focus of our study is (i) how to quantify, at least in quasistatic limit, the genuinely curvilinear interfaces of the cylindrical hyperlens in a radius-dependent effective medium theory (EMT) and (ii) if this EMT could improve the performance of the cylindrical hyperlens. The initial analysis indicated that a nonperiodic distribution of thicknesses of the lens layers is required for the improved design. Detailed simulations prove superior functioning of the lens designed with the improved approach versus the designs with the uniformly thick layers if the same number and starting order of layers is taken. The numerical analysis is built on the solution of the wave equation in piecewise homogeneous cylindrical coordinates, which has been widely detailed in literature. Our study is using the scalar wave equation; the electric field is assumed to be two-dimensional and the magnetic field is the purely scalar quantity. For TM polarization the magnetic field (parallel to $\mathbf{H}$) can be expanded in azimuthal Fourier modes, $h(\rho, \phi) = \Sigma h_m(\rho) \exp i m \phi$ and the wave equation for the $m$th mode reduces to

$$\varepsilon_\rho \rho^{-1} (\varepsilon_\rho^{-1} h_m')' + [k^2 \varepsilon_\mu \rho_z - (m\rho)^2] h_m = 0. \quad (1)$$

Here the physical Cartesian coordinates $(x, y, z)$ are defined through the cylindrical coordinates $(\rho, \phi, z)$ as $x = \rho \cos \phi$, $y = \rho \sin \phi$, and $z = z$; the prime corresponds to the radial derivative $\partial / \partial \rho$, $\varepsilon_\rho$ and $\varepsilon_\phi$ are the only nonzero diagonal components of the anisotropic permittivity tensor, and $k$ is the wavenumber of free-space. While the ideal and simplified hyperlens paradigms are based on smooth scaling transformations of Eq. (1), the proposed design of a manufacturable discrete-layered cylindrical hyperlens does not yet imply quantum size effects and other complications at the metal-dielectric interfaces, and thus each layer is taken to have isotropic dielectric function ($\varepsilon_\rho = \varepsilon_\phi$). The transformation of field data within each cylindrical layer to another place is accomplished by representing the field as a summation of spatial harmonics. The harmonic coefficients are determined by matching the fields at the interfaces with the expansion solution of the wave equation. Once these coefficients are found, the field is computed everywhere by using the sums of spatial harmonics. Making use of the cylindrical mode superposition, the source field and scattered fields are decomposed into backward (inbound) and forward (outbound) modes. The transfer matrices are deduced directly from Maxwell equations and are cast in a spatial spectral form and then in matrix relations. Test problems are simulated using our ad hoc online tool, where the hyperlens may consist of any number of layers bounded by infinite concentric cylindrical surfaces. The material properties of the media layers are utilizing the material optical database where the materials are allowed to be dispersive, embracing polaritonic (plasmonic or phononic) media. Consider a classical hyperlens [a circular cylindrical multilayered structure comprising $l_{\text{max}}$ cylindrical interfaces of lamella (Fig. 1)]. The axes of concentric cylindrical surfaces are assumed to coincide with the $z$-axis of a common cylindrical coordinate system $(\rho, \phi, z)$. Each interface boundary with a radius $\rho_l$ separates two homogeneous media with the constant permeability $\mu = 1$ and a pair of dielectric functions, $\varepsilon_l$ and $\varepsilon_{l+1}$. In TM formulation for a fixed free-space wavelength $\lambda$, the scalar magnetic field for a given $l$th interface is described by four components: the transmitted ($h^{21}_l$) and reflected ($h^{21}_l$) field at

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**FIG. 1.** (Color online) (a) Geometry of the concentric circular domains. (b) Example of the test source inside the zoomed internal region of the lens.
the inner side of the boundary, \( h_{2i}^{l} \) and \( h_{2i}^{k} \) field at the inner side of the boundary
\[
h_{2i}^{l} = \sum j_{m}^{l} e^{i m \phi}, \quad h_{2i}^{k} = \sum j_{m}^{k} e^{i m \phi},
\]
and the transmitted \( (h_{2i}^{l+1}) \) and reflected \( (h_{2i}^{r+1}) \) fields at the outer side of the interface boundary
\[
h_{2i}^{l+1} = \sum j_{m}^{l+1} e^{i m \phi}, \quad h_{2i}^{r+1} = \sum j_{m}^{r+1} e^{i m \phi},
\]
where factors \( j_{m}^{\pm 1} = j_{m}^{l+1} / j_{m}^{l} \), \( j_{m}^{\pm} = b_{m}^{l+1} / b_{m}^{l} \) can be defined using the forward \( \{f_{m}\} \) and backward \( \{b_{m}\} \) radial functions are the Hankel and Bessel cylindrical functions of the first kind. Thus, the even and odd terms are given by
\[
 J_{m}^{l+1}(q_{l}) = \frac{\partial J_{m}^{l}(q_{l})}{\partial q_{l}}, \quad J_{m}^{r+1}(q_{l}) = \frac{\partial J_{m}^{r}(q_{l})}{\partial q_{l}}.
\]
(3)

For testing, the incident field can be, for example, generated by an expansion of the step-function, forcing the incident field to be unity at an arc segment with a constant radius \( r_{i} \) and angular dimensions \( \phi_{i}=[\phi_{i1}, \phi_{i2}] \).\[\text{Fig. 1b}(\text{b})\]. If we apply the complete version of the same addition theorem in \( h_{l} = H_{l}^{1}(k_{n} r_{i}) m_{i} \), the incident field is given either by
\[
h_{l} = \sum_{m}^{l} e^{i m \phi} (\text{for sources distributed inside the lens, } \rho_{i} > r_{i}) \quad \text{or} \quad h_{l} = \sum_{m}^{l} e^{i m \phi} (\text{for external sources, } \rho_{i} < r_{i}).
\]
Then, for \( \rho_{i} < r_{i} \), \( \psi_{m} \equiv \psi_{m}^{l} \equiv \psi_{m}, \) as well as \( \psi_{m}^{\phi} \equiv \psi_{m}^{l} \equiv \psi_{m} \), where \( m > 0 \), \( \psi_{m} = (e^{-\omega_{d}^{m} d_{i}-2-\omega_{m}^{m} d_{i}}, \phi_{m}^{l} = \phi_{m}^{l} = \phi_{m}^{l} = \phi_{m}^{l}) \), otherwise, \( \psi_{m} = 1 \). In practice of designing engineered optical spaces, a smooth ideal distribution is approximated by a few elemental materials. Thus normally, engineered optical spaces, a smooth ideal distribution is approximated by a few elemental materials. Thus, the even and odd terms are given by
\[
= \frac{1}{\partial q_{l}}.
\]
(2)

Field-matching boundary conditions at \( l \)th interface are given by
\[
h_{l+1}^{l+1} + h_{l-1}^{r+1} = h_{l+1}^{r+1} + h_{l-1}^{l+1}, \quad \frac{1}{\partial l}.
\]
(3)

\[\text{FIG. 2. (Color online) (a) Input magnetic field intensity at } \rho_{i} \text{ used for all tests. (b) Comparison of the nonuniform distribution of } \rho_{i} \text{ obtained from Eq. (9) vs the uniform distribution; } l_{\text{max}}=34 \text{ has been taken for this example.}\]

Thus, \( \frac{1}{\partial l} \) can be rewritten using a matrix notation \( v_{m}^{l} = w_{m}^{l} v_{m+1,l} \) with the components of vector \( v_{m} \) being \( (v_{m})_{0} = r_{m} \) and \( (v_{m})_{1} = r_{m} \) while the elements of matrix \( w_{m} \) are given by \( w_{m}^{l,1} = \frac{1}{j_{m}^{l}(q_{l})} \), \( w_{m}^{l,0} = \frac{1}{j_{m}^{l}(q_{l})} \), \( w_{m}^{l,1} = \frac{1}{j_{m}^{l}(q_{l})} \), \( w_{m}^{l,0} = \frac{1}{j_{m}^{l}(q_{l})} \), and \( w_{m}^{l,1} = \frac{1}{j_{m}^{l}(q_{l})} \).

Finally, cascading elementary layers (Fig. 1) gives \( v_{m}^{l} = \Phi^{l} v_{m}^{l-1} \), \( w_{m}^{l} = \Phi^{l} w_{m}^{l-1} \), \( n_{m}^{l} = \lambda_{m}^{l} n_{m}^{l-1} \), \( l_{m}^{l} = l_{m}^{l-1} n_{m}^{l-1} \), and \( n_{m}^{l} = \lambda_{m}^{l} n_{m}^{l-1} \).

The performance of the hyperlens is tested for two designs, proposed for two elemental materials SiC and ZnSe to be operational at a wavelength of \( \lambda = 11.061 \mu m \), with \( e_{\text{SiC}} = 5.027+0.271i \), and \( e_{\text{ZnSe}} = 5.597 \). Note that at this wavelength both dielectric functions satisfy the condition \( \text{Re}(e_{\text{SiC}}) + \text{Re}(e_{\text{ZnSe}}) = 1 \), providing good balance of elemental materials in the structure. The first design (lens A) is arranged with uniformly spaced interfaces, which in accord with Eqs. (5) and (6) would give constant values of \( \text{Re}(e_{\phi,l}) = 1 \) and \( \text{Re}(e_{\phi,l}) = 10 \). Though as shown in Fig. 2, lens A gives staggered values of \( e_{\phi,l} \) and \( e_{\phi,l} \).

If calculated for the same uniform square layers (SiC–ZnSe or SiC–SiC) but without fixing \( \text{Re}(e_{\phi,l}) \), this design rule defines the following three term recurrence for interface radii:
\[
\rho_{m+1} = \rho_{m}(\rho_{i}^{l-1} / \rho_{i}^{l})^{\text{Re}(e_{\phi,l}) / \text{Re}(e_{\phi,l}) = 1}.
\]

Thus for a given number of layers \( l_{\text{max}} \) in lens B, the entire design is achieved through adjusting the sequence of radii in order to fit the outer radius \( \rho_{i}^{l_{\text{max}}} \).

While compared, both lens designs share a common set of dimensions, \( \rho_{i} = \lambda, \) and \( l_{\text{max}} = 3\lambda, \) and the same number of elemental layers. It has been observed that a lens with non-uniform thicknesses of layers, lens B always gives a better resolution in comparison with lens A with the same number of layers, and the best performance for lens B is achieved with the odd number of layers with the inner layer made of metal (SiC–ZnSe sequence). For example, Fig. 3 compares imaging with lens A versus lens B; the normalized distance in Fig. 3 is defined as \( d_{s} = l_{\text{max}}^{l} \phi / \lambda \). For both lenses, the test image is obtained with the best-performing order of elementary layers (SiC–ZnSe for \( l_{\text{max}} = 31 \) and ZnSe–SiC for \( l_{\text{max}} = 32 \)). Figures 3(b) and 3(d) show that lens B is already working quite well even for such small numbers of layers,
while lens A is barely resolving the image [Fig. 3(c)] or does not resolve it at all [Fig. 3(d)].

In the past few years, an improved EMT has been applied to obtain nonlocal corrections to the quasistatic EMT of anisotropic materials made of lamellar structures with subwavelength-thick layers. Thus, a near term work could include the incorporation of nonlocal adjustments for cylindrical system in a way much similar to the techniques introduced for planar structures.

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